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Optimal Radially Accelerated Interplanetary Trajectories

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Interplanetary trajectories for a propulsion system providing a continuous outward radial thrust that varies according to the inverse square of heliocentric distance are investigated. This type of radial acceleration regime is realized by sun-facing solar sails and minimagnetospheric plasma propulsion, which allow the acceleration magnitude to be modulated in flight. Formulating the interplanetary trajectories as an optimal control problem, the escape from the solar system is studied, considering the maximization of the terminal orbital energy, while conserving the orbital angular momentum. The achievable hyperbolic excess velocity is studied in terms of the available maximum radial acceleration and the transfer angle. The inclusion of the Earth gravity assist for the escape from the solar system is shown to provide a more efficient means of achieving escape at the expense of flight time. Transfer between circular orbits is similarly realized by a combination of radial acceleration propulsion and planetary gravity assist, in which the radial acceleration acts as a control of the orbital energy and the planetary gravity assist acts as a control of the angular momentum.

Nomenclature

E = nondimensional orbital energy
H = Hamiltonian function
h = nondimensional orbital angular momentum
p = derivative of u with respect to polar angle
r_{EGA} = swingby distance of Earth gravity assist
r_{JGA} = swingby distance of Jupiter gravity assist
u = inverse nondimensional radial distance
v = magnitude of nondimensional velocity
v_{EGA} = relative velocity with Earth at Earth gravity assist
v_{JGA} = relative velocity with Jupiter at Jupiter gravity assist
β_{EGA} = turn angle of relative velocity by Earth gravity assist
β_{JGA} = turn angle of relative velocity by Jupiter gravity assist
ε = nondimensional acceleration
$\varepsilon_{\mathrm{Max}}$ = maximum nondimensional acceleration defined at Earth distance
θ = angular position (polar coordinate)
λ_p = adjoint variable associated with p
λ_u = adjoint variable associated with u
μ_{Earth} = gravity constant of Earth, 398,600 $\mathrm{km}^3/\mathrm{s}^2$
μ_{Jupiter} = gravity constant of Jupiter, 398,600 × 317.83 $\mathrm{km}^3/\mathrm{s}^2$
ρ = nondimensional radial distance (polar coordinate)
ρ_{Jupiter} = nondimensional Jupiter heliocentric distance
τ = nondimensional time
υ = multiplier associated with u
ϕ = performance index
ψ = constraining function of the terminal state

Subscripts

aft = quantities achieved by accelerating beyond the final instant $t = t_f$
f = quantities at the final instant $t = t_f$ (i.e., crossing Earth's heliocentric distance)
inf = quantities at infinite distance
0 = quantities at the starting instant $t = 0$ (i.e., Earth's orbit)

Superscripts

$-$ = quantities before planetary gravity assist
$+$ = quantities after planetary gravity assist
$'$ = derivative with respect to polar angle

Introduction

THE classical problem of spacecraft trajectory under a continuous radial acceleration has been investigated by many researchers in the past.[1−11] Tsien,[1] Irving,[2] and Battin[3] showed that there exists a critical value of constant radial acceleration above which the spacecraft will achieve escape conditions. Boltz[4] treated the case of radial acceleration inversely proportional to the square of the distance from the central body using an equation of motion described by the velocity and the flight path angle. Prussing and Coverstone-Carroll[5] provided a nonlinear radial spring interpretation for the constant radial acceleration problem based on an energy-well approach. Akella[6] showed that for the constant radial acceleration case, although the time intervals for the outbound and inbound trajectories are identical, the trajectories themselves are very different. Broucke and Akella[7] described the general types of solutions for the continuous constant outward radial acceleration problem, employing numerical integration and concepts such as the theory of periodic orbits and Poincare's characteristic exponents as a basis. Broucke[8] reviewed several results related to the classical problem of two-dimensional motion of a particle in the field of a central force proportional to a real power of the distance. Trask et al.[9] investigated several outer-planet missions (i.e., Earth–Mars, Earth–Jupiter, and Earth–Saturn transfers), where the radial thrust propulsion system is used only for an initial period. The authors used a nonlinear programming method to minimize the impulsive velocity change required to circularize the orbit, by varying the radial thrust propulsion burn time. They also considered a Pluto mission with the first burn to Venus, gravity assist at Venus, and then a burn to Pluto. McInnes[10] extended the classical two-body problem to consider the addition of a modulated, radial, inverse-square force, and explored the families of orbits by forward integration and an inverse approach that allows orbits defined a priori. McInnes[10] also defined a switching strategy that realizes the motion along an escape ladder in the semimajor-axis-eccentricity plane, providing the orbital energy gain while conserving the orbital angular momentum. Yamakawa[11] devised a guidance strategy for radially accelerated trajectories based on the optimum spacing rule.

In this paper, spacecraft trajectories in the case of a continuous outward radial acceleration that diminishes as the inverse square of heliocentric distance are treated as an optimal control problem. Interplanetary trajectories starting from an Earth orbit are investigated, where the terminal orbital energy is maximized. The results are applicable for propulsion systems such as solar-sail propulsion, using the flux of photons,[12] and minimagnetospheric plasma propulsion

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(M2P2), which deflects the solar-wind plasma.[13,14] For the solar-sail case, a passively controlled sun-pointing sail with a slightly conical form is assumed, where the apex is directed sunward.[10] Normally, solar sails can be articulated to provide transverse and radial thrust, but a sun-facing attitude might be preferred for a very large and/or gossamer sail. As the flux of photons intercepted by the solar sail and the power available to drive the M2P2 propulsion system decrease in proportion to the inverse square of heliocentric distance, these two propulsion systems generate essentially radial forces that vary according to the inverse square of heliocentric orbit.[10] However, as the effective area of the solar sail can be altered, and the thrust of the M2P2 system can be changed by varying the size of the bubble of the magnetic field, the magnitude of the acceleration can be varied in flight. Using this concept, interplanetary trajectories are formulated as an optimal control problem. The treatment considers the attainable hyperbolic excess velocity determined by the radial acceleration magnitude and the transfer angle. Moreover, a combination of radial acceleration propulsion and planetary gravity assist is employed to realize escape from the solar system and transfer between coplanar circular orbits.

Statement of the Problem

The present analysis considers the motion of a spacecraft under the influence of its thrust and the gravitational attraction of a central body, namely, the sun. The acceleration caused by the thrust of the spacecraft is confined to the outward radial direction, ensuring that the trajectory remains in a plane and can therefore be described by two-degree-of-freedom equations of motion. If the acceleration is in the outward radial direction and decreases as the inverse square of heliocentric distance, the nondimensional equations of motion for the spacecraft in polar coordinates are given by[10,11]

$$\frac{\mathrm{d}^2\rho}{\mathrm{d}\tau^2} = \rho\left(\frac{\mathrm{d}\theta}{\mathrm{d}\tau}\right)^2 - (1-\varepsilon)\frac{1}{\rho^2} \tag{1}$$

$$\frac{\mathrm{d}}{\mathrm{d}\tau}\left(\rho^2\frac{\mathrm{d}\theta}{\mathrm{d}\tau}\right) = 0 \tag{2}$$

The variables are normalized by the radius of the Earth's orbit, the corresponding circular velocity, and the magnitude of the gravitational attraction of the sun at the Earth's radial distance. The period of the Earth's orbit is 2π, and the reference time unit is 58.132 days. The reference distance, velocity, and acceleration units are $1.49597870 \times 10^{11}$ m, 29,784 m/s, and 0.00593 m/s^2, respectively. Equation (2), describing the conservation of angular momentum h, can then be integrated as follows:

$$\frac{\mathrm{d}\theta}{\mathrm{d}\tau} = \frac{h}{\rho^2} \tag{3}$$

Substitution of this equation into Eq. (1) yields

$$\frac{\mathrm{d}^2\rho}{\mathrm{d}\tau^2} = \frac{h^2}{\rho^3} - (1-\varepsilon)\frac{1}{\rho^2} \tag{4}$$

which can be rewritten as

$$\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}\rho}\left(\frac{\mathrm{d}\rho}{\mathrm{d}\tau}\right)^2 = \frac{h^2}{\rho^3} - (1-\varepsilon)\frac{1}{\rho^2} \tag{5}$$

The radial velocity $\mathrm{d}\rho/\mathrm{d}\tau$ is then given by integrating the preceding equation in terms of ρ from ρ_0 to ρ as follows:

$$\frac{1}{2}\left(\frac{\mathrm{d}\rho}{\mathrm{d}\tau}\right)^2 = \frac{1}{2}\left(\frac{\mathrm{d}\rho}{\mathrm{d}\tau}\right)_0^2 + \frac{h^2}{2}\left(-\frac{1}{\rho^2} + \frac{1}{\rho_0^2}\right) + (1-\varepsilon)\left(\frac{1}{\rho} - \frac{1}{\rho_0}\right) \tag{6}$$

The equations of motion are further transformed into a simple form by introducing a new variable $u = 1/\rho$ (Ref. 10). Changing the independent variable from nondimensional time τ to polar angle θ, equations of motion [Eqs. (3) and (4)] are then transformed to

$$u'' + u = 1 - \varepsilon \tag{7}$$

where the prime indicates a derivative with respect to polar angle θ. With this equation, the present analysis treats a radially accelerated trajectory starting from a circular orbit and returning to the same heliocentric distance, aiming at maximizing the terminal orbital energy.

Optimal Control

The radially accelerated trajectory starting from Earth's orbit and returning to Earth's heliocentric distance is treated as an optimal control problem in which the terminal outward radial velocity $(\mathrm{d}\rho/\mathrm{d}\tau)_f$ is maximized, or, equivalently, the terminal inverse radial velocity $(u')_f$ is minimized. This corresponds to the problem of maximizing the orbital energy while conserving the angular momentum. The objective is thus to obtain the optimal acceleration program that minimizes $(u')_f$ within upper and lower bounds $0 \le \varepsilon \le \varepsilon_{\mathrm{Max}}$. The optimal radial acceleration program is based on the optimal control theory and the classical calculus-of-variation approach.[15] The inverse radial distance dynamics is governed by the following dynamical equations via Eq. (7):

$$u' = p \tag{8}$$

$$p' = -u + 1 - \varepsilon \tag{9}$$

To apply the theory of optimal control, a Hamiltonian function is defined:

$$H = \lambda_u p + \lambda_p(-u + 1) - \lambda_p\varepsilon \tag{10}$$

where the dynamical equations for the adjoint variables are defined as follows:

$$\lambda_u' = \lambda_p \tag{11}$$

$$\lambda_p' = -\lambda_u \tag{12}$$

Pontryagin's minimum principle[15] requires that H be minimized with respect to all admissible controls at all times, which leads to the optimal acceleration program

$$\varepsilon = 0 \qquad \text{when} \qquad \lambda_p < 0$$

$$\varepsilon = \varepsilon_{\mathrm{Max}} \qquad \text{when} \qquad \lambda_p > 0 \tag{13}$$

The constraining function ψ of the terminal state, which ensures that the spacecraft returns to Earth's distance, is given by

$$\psi = u_f - 1 = 0 \tag{14}$$

The performance index ϕ, defined at the terminal state, aimed at maximizing the terminal radial velocity, is expressed as

$$\phi = u_f' = p_f \tag{15}$$

Transversality conditions[15] are derived from Eqs. (14) and (15) as follows:

$$(\lambda_u)_f = \upsilon \tag{16}$$

$$(\lambda_p)_f = 1 \tag{17}$$

Finally, the initial conditions of the state variables are

$$u_0 = 1 \tag{18}$$

$$p_0 = 0 \tag{19}$$

The four linear differential equations (8), (9), (11), and (12) are to be solved subject to the four boundary conditions (16), (17), (18), and (19), with the choice of multiplier υ available to satisfy the additional boundary condition (14). The radial acceleration control ε is determined in terms of λ_p from Eq. (13).

Escape from the Solar System

Table 1 summarizes the characteristics of optimum radially accelerated trajectories, where the terminal radial velocity $\dot{\rho}_f$ upon return to the Earth's heliocentric distance is maximized. The design parameters are $\varepsilon_{\mathrm{Max}}$ and θ_f considered from an Earth orbit to the Earth's heliocentric distance. The unit of transfer angle θ_f is the number of revolutions around the sun. The flight time τ_f is obtained by integrating the following equation derived from Eq. (3):

$$\frac{\mathrm{d}\tau}{\mathrm{d}\theta} = \frac{1}{u^2} \tag{20}$$

In the table, the unit of τ_f is Earth's orbital period (Earth year). The attainable terminal radial velocity increases with the transfer angle, and an escape from the solar system is finally achieved. The hyperbolic excess velocity $\dot{\rho}_{\mathrm{inf}.f}$ is shown when the trajectory achieves a hyperbolic state at τ_f (cases 1-2 and 1-4 in Table 1). Although the terminal point of the present analysis is assumed to be the point of crossing with the Earth's heliocentric distance, the spacecraft can be further accelerated continuously even after passing that distance. Table 1 also shows the hyperbolic excess velocity $\dot{\rho}_{\mathrm{inf.aft}}$ achieved by accelerating the spacecraft after τ_f, which is given by

$$\dot{\rho}_{\mathrm{inf.aft}} = \sqrt{\dot{\rho}_f^2 - 2 + h^2 + 2\varepsilon} \tag{21}$$

derived from Eq. (6).

Figure 1 shows the trajectory plot corresponding to case 1-1, where the acceleration region is plotted in bold. Under the variable radial acceleration scheme, the first and second aphelion distances are 1.39 and 2.72, respectively, and the terminal osculating aphelion distance is 5.01 at the Earth's heliocentric distance at τ_f. If the spacecraft is simply accelerated from the Earth's distance to the aphelion point, the attainable aphelion distance is only 1.67, which can be derived from the equation $1/(1 - 2\varepsilon_{\mathrm{Max}})$ (Ref. 4). This shows that the optimum thrusting provides a rather larger aphelion distance than the simple half-arc one. The spacecraft is accelerated continuously even after τ_f and finally attains a hyperbolic trajectory.

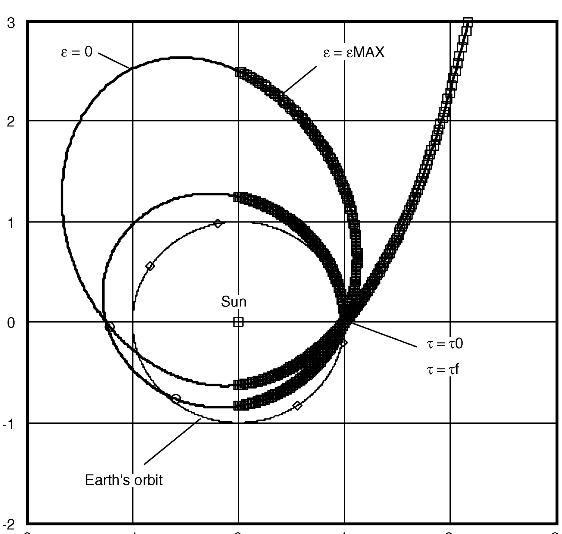


Fig. 1 Optimal radially accelerated trajectory plot ($\varepsilon_{\mathrm{Max}} = 0.2$, $\theta_f = 2.0$ rev).

Table 1 Optimal radially accelerated trajectories

Case	$\varepsilon_{\mathrm{MAX}}$	θ_f, rev	τ_f, yr	$\dot{\rho}_0$	$\dot{\rho}_f$	$\dot{\rho}_{\mathrm{inf}.f}$	$\dot{\rho}_{\mathrm{inf.aft}}$
1-1	0.2	2.0	3.43	0.00	0.80	——	0.20
1-2	0.2	2.8	8.00	0.00	1.00	0.06	0.64
1-3	0.4	1.0	1.88	0.00	0.80	——	0.66
1-4	0.4	1.2	5.77	0.00	1.13	0.53	1.04

Tables 1 indicates that, although direct escape, accelerating continuously from an initial Earth orbit to reach a hyperbolic velocity requires $\varepsilon_{\mathrm{Max}} \geq 0.5$ (Ref. 4); the optimal variable radial-acceleration solution thus reduces the required radial acceleration at the expense of flight time.

Effect of Earth Gravity Assist

Table 2 lists some specific cases where the difference between the transfer angle (unit: revolution) and the flight time (unit: Earth's orbital period) is a whole integer. In these cases, the spacecraft returns to Earth with a positive radial velocity, and an Earth gravity assist (EGA) can be realized. The effects of the EGA are discussed here in terms of variations in the orbital energy and the angular momentum. The EGA can turn the relative velocity with respect to the Earth by an angle of[16]

$$\beta_{\mathrm{EGA}} = 2\sin^{-1}\left[1\big/\left(1 + r_{\mathrm{EGA}} v_{\mathrm{EGA}}^2 \big/ \mu_E\right)\right] \tag{22}$$

where the relative velocity v_{EGA} is equivalent to the spacecraft radial velocity prior to the EGA because the spacecraft circumferential velocity is equivalent to that of the Earth. The variations in the orbital energy and the angular momentum can then be calculated easily as follows:

$$\Delta E = \frac{(v^+)^2}{2} - \frac{(v^-)^2}{2} = \dot{\rho}_f \sin\beta_{\mathrm{EGA}} \tag{23}$$

$$\Delta h = \left(\frac{\rho\,\mathrm{d}\theta}{\mathrm{d}\tau}\right)^+ - \left(\frac{\rho \mathrm{d}\theta}{\mathrm{d}\tau}\right)^- = \dot{\rho}_f \sin\beta_{\mathrm{EGA}} \tag{24}$$

Equations (23) and (24) show that the EGA enhances the orbital energy and angular momentum, the latter of which cannot be controlled solely by radial acceleration propulsion. Table 2 shows how the EGA works in favor of the hyperbolic excess velocity $\dot{\rho}_{\mathrm{inf.aft}}$, including the effect of subsequent radial acceleration after the EGA. The swingby altitude is assumed to be 1000 km ($r_{\mathrm{EGA}} = 7378$ km). Table 2 also lists the angular momentum and orbital energy values prior to and after the EGA. The velocity turn angle by the EGA is about 10 deg, and the hyperbolic excess velocity increases by approximately 0.2 (cases 2-2–2-4).

Application to Transfer Between Coplanar Circular Orbits

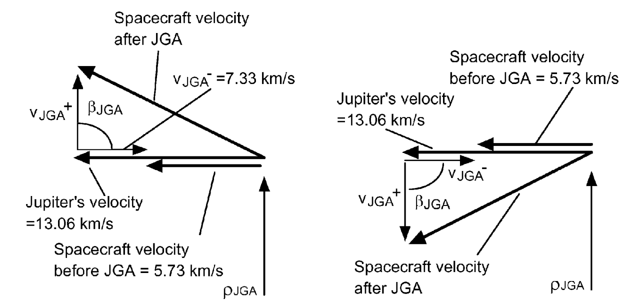
The radial-acceleration and planetary gravity-assist concepts are combined here to realize a transfer between circular orbits. The transfer from an Earth orbit to a circular orbit at Jupiter's distance is taken as an example. A sequence consisting of three phases is considered: radially accelerated Earth–Jupiter transfer phase, Jupiter gravity assist (JGA), and radially accelerated JGA-circular-orbit transfer phase. For the Earth–Jupiter phase, a radial acceleration is applied to increase the orbital energy while conserving the angular momentum.

Table 2 Optimal radially accelerated Earth-return trajectories

			Parameters									
			Without EGA			EGA effect				With EGA		
Case	$\varepsilon_{\mathrm{Max}}$	θ_f, rev	τ_f, yr	$\dot{\rho}_0$	$\dot{\rho}_f$	$\dot{\rho}_{\mathrm{inf.aft}}$	h^-	E^-	β_{EGA}	gh^+	E^+	$\dot{\rho}_{\mathrm{inf.aft}}$
2-1	0.2	1.943	2.943	0.00	0.73	——	1.00	−0.23	11.8	1.15	−0.08	0.48
2-2	0.2	2.051	4.051	0.00	0.86	0.38	1.00	−0.13	8.7	1.13	−0.03	0.64
2-3	0.2	2.114	5.114	0.00	0.93	0.51	1.00	−0.07	7.5	1.12	+0.05	0.71
2-4	0.2	2.173	6.173	0.00	0.97	0.58	1.00	−0.03	7.0	1.11	+0.09	0.76

Table 3 Optimal radially accelerated JGA-circular orbit-transfer trajectories

Case	$\varepsilon_{\mathrm{Max}}$	θ_f, rev	τ_f, yr	$\dot{\rho}_0$	$\dot{\rho}_f$
3-1	0.28	0.75	1.97	0.56	0.00
3-2	0.40	0.90	1.28	−0.56	0.00



a) Outward turn (case 3-1 in Table 3) **b) Inward turn (case 3-2 in Table 3)**

Fig. 2 JGA velocity vector diagram.

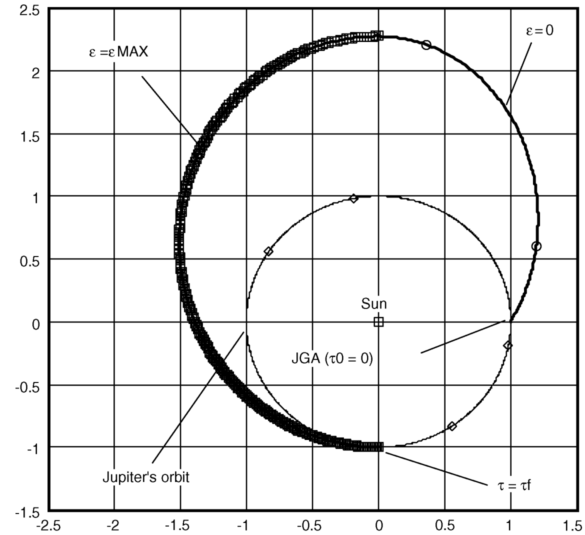


Fig. 3 Optimal radially accelerated JGA-circular-orbit-transfer trajectory (case 3-1 in Table 3).

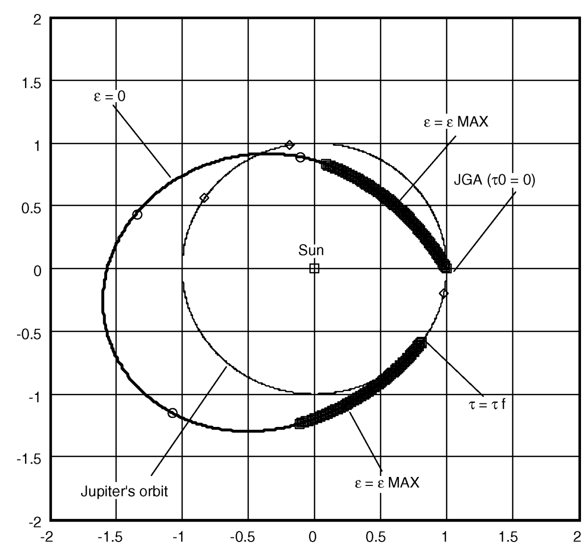


Fig. 4 Optimal radially accelerated JGA-circular-orbit trajectory (case 3-2 in Table 3).

The objective of the JGA is to enhance the nondimensional angular momentum from that of the Earth (i.e., 1) to that of Jupiter (i.e., $\rho_{\mathrm{Jupiter}} \times \sqrt{(1/\rho_{\mathrm{Jupiter}})} = \sqrt{(\rho_{\mathrm{Jupiter}})}$, $\rho_{\mathrm{Jupiter}} = 5.2026$). This is equivalent to enhancing the nondimensional circumferential velocity by $\sqrt{(1/\rho_{\mathrm{Jupiter}})} - 1/\rho_{\mathrm{Jupiter}} = 0.24$ (i.e., 7.33 km/s) at Jupiter's distance. The purpose of the final JGA-circular-orbit transfer phase is to change the orbital energy to that equivalent to Jupiter's orbital motion, while maintaining the angular momentum, in order to achieve a circular orbit.

Here, it is assumed that the aphelion distance of the first Earth–Jupiter transfer phase is Jupiter's distance, which requires $\varepsilon_{\mathrm{Max}} = 0.4$ (Ref. 4). The radial velocity of the spacecraft at Jupiter's distance is then zero, and the relative velocity v_{JGA} and velocity turn angle β_{JGA} are 7.33 km/s and 90 deg, respectively (Fig. 2). Note that two cases (i.e., outward and inward velocity turns) can be considered for the JGA to achieve an angular momentum equivalent to that of Jupiter. The equation[16]

$$\beta_{\mathrm{JGA}} = 2\sin^{-1}\left[1\Big/\left(1 + r_{\mathrm{JGA}} v_{\mathrm{JGA}}^2 \Big/ \mu_{\mathrm{Jupiter}}\right)\right] \tag{25}$$

yields $r_{\mathrm{JGA}} = 976{,}672$ km, equivalent to 13.7 Jupiter radii.

The final JGA-circular-orbit transfer phase can be solved as an optimal control problem using Eqs. (8) and (9). The variables in this case are normalized using the radius of Jupiter's orbit, the corresponding circular velocity, and the magnitude of the gravitational attraction of the sun at Jupiter's distance. The period of Jupiter's orbit is 2π, and the reference time unit is 689.840 days. The reference distance, velocity, and acceleration units are $5.2026 \times 1.49597870 \times 10^{11}$ m, 13,058 m/s, and 0.000219 m/s^2, respectively. The starting instant τ_0 corresponds to the JGA, and the final instant τ_f corresponds to the arrival at a circular orbit at the distance of Jupiter. The radial acceleration works to decrease the terminal radial velocity in the outward velocity turn case and increase the terminal radial velocity in the inward turn case (Fig. 2). The initial condition for the radial velocity is ±7.33 km/s (Fig. 2), which is nondimensionalized to ±0.56 using Jupiter's velocity of 13.06 km.

Table 3 summarizes the characteristics of JGA-circular-orbit transfer trajectories for the outward and inward velocity turn cases. Appropriate combinations of θ_f and $\varepsilon_{\mathrm{Max}}$ are sought to nullify the terminal radial velocity $\dot{\rho}_f$ and to realize a circular orbit. Every variable is nondimensionalized based on Jupiter's orbit. Figures 3 and 4 show the corresponding trajectory plots from the JGA to the circular orbit insertion. The results show that circular-to-circular orbit transfer can be achieved by a combination of radial acceleration propulsion and planetary gravity assist. Realistic transfers, which include gravity assists, will require modest plane change, unless orbit phasing can be used to perform gravity assists at the nodes of the transfer orbit.

Conclusions

The orbital motion under a continuous outward inverse square radial acceleration was investigated assuming a propulsion system in which the magnitude of the radial acceleration can be modulated, such as sun-facing solar sails and minimagnetospheric propulsion. Interplanetary trajectories were thus formulated as an optimal control problem. Escape trajectories were examined assuming the maximization of the orbital energy, while conserving the angular momentum, and the attainable hyperbolic excess velocity was determined based on the maximum radial acceleration magnitude and the transfer angle. A direct escape requires a nondimensional radial acceleration of 0.5, while the optimal solution provides a dramatic reduction in the required maximum radial acceleration at the expense of flight time. The combination of radial acceleration propulsion and Earth gravity assist was found to be an efficient means of realizing escape from the solar system. The Earth gravity assist has the effect of either reducing the required acceleration capability or increasing the hyperbolic excess velocity. Transfer between circular orbits using a planetary gravity assist was also investigated, and it was shown that transfer from an Earth orbit to a circular orbit at

Jupiter's distance is possible through the exploitation of a Jupiter gravity assist to control the angular momentum in combination with a radial acceleration propulsion to control the orbital energy.

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